CS 9

Week 4 Problems

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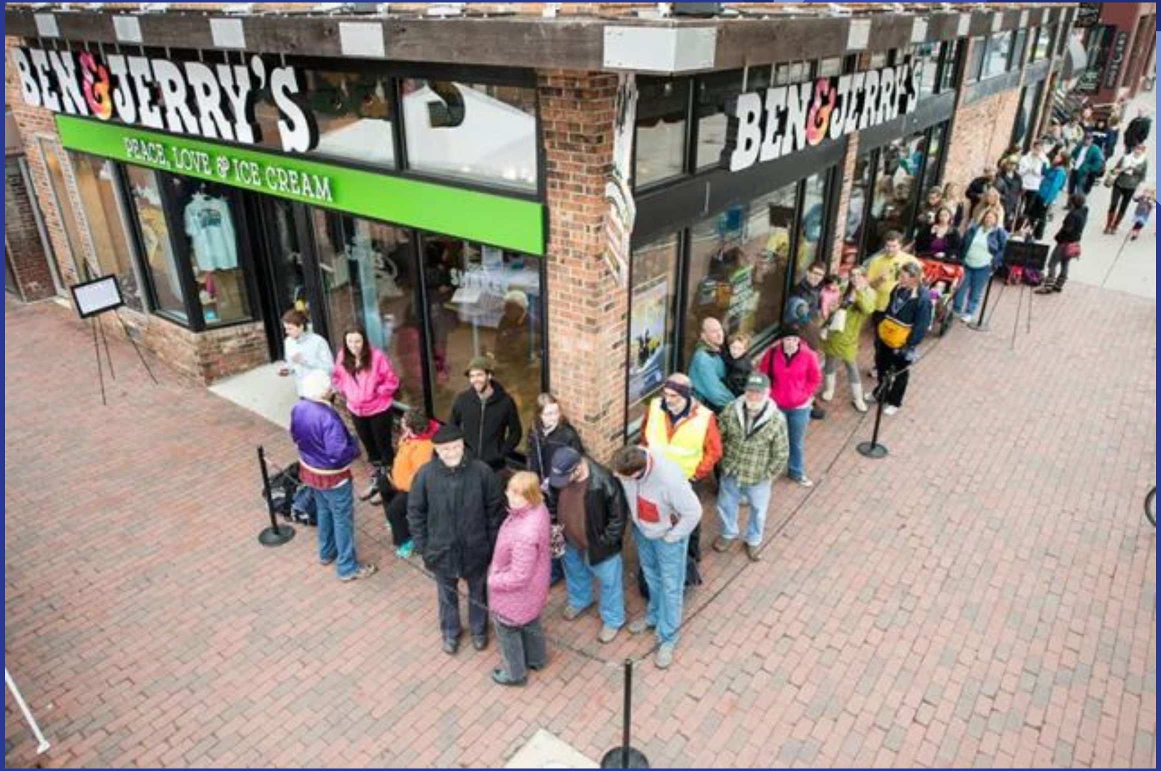
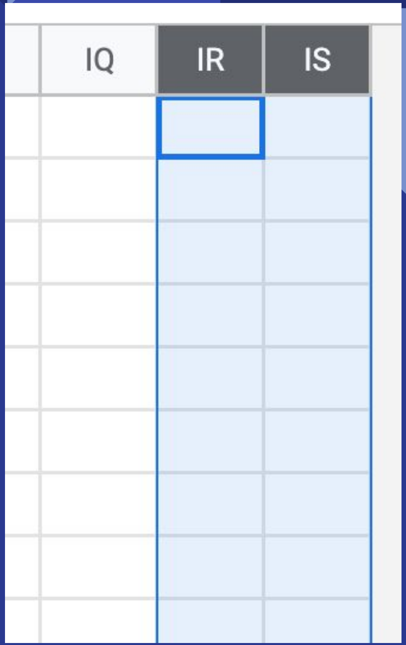


⭐ Problem 4-1: Conseculumns

- In spreadsheets, columns are labeled A, B, …, Z, then AA, AB, …, ZZ, and so on.
- Write a function that, given a string (length up to 10000), determines whether that string consists of exactly two consecutive column labels, in order. (And be careful!)



`HI`: True

`XERXES`: True

`XW`: False

`ABABC`: False

⭐⭐ Problem 4-2: Trendy Dessert

- Some new dessert place just dropped in Palo Alto, and there's a huge line.
  - Is it really that good though?
    - OK it's probably pretty good

- There are K servers (2 <= K <= 1000); the i-th server takes K_i minutes to help a customer, then opens up for the next one. Customers go to the lowest-numbered available server.

- The store just opened. You are position N in line (1 <= N <= 10^9). **Which server will help you?**

Example

e.g., $K_1 = 20$, $K_2 = 10$, $K_3 = 15$: three servers
$N = 6$: you are customer 6 in line

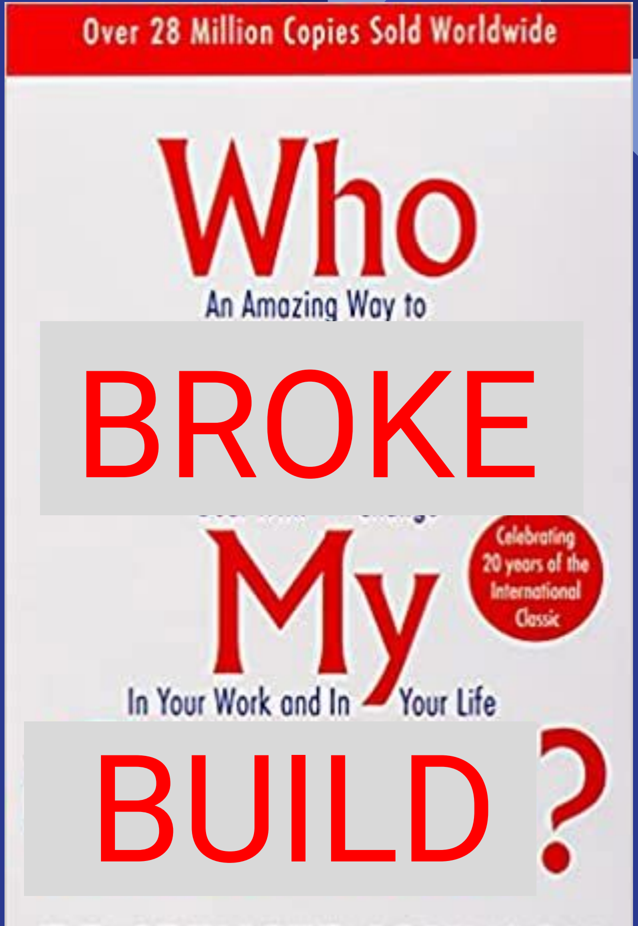
0 min:
Store opens. Customers 1, 2, and 3 go to servers 1, 2, and 3.
10 min:
Server 2 finishes. Customer 4 goes to server 2.
15 min:
Server 3 finishes. Customer 5 goes to server 3
20 min:
Servers 1 and 2 finish. You go to server 1. Customer 7 goes to server 2.

⭐⭐ Problem 4-3: Average Joe

- You want to cast the lead of a movie called "Average Joe".
- A total of K * N actors have applied.
- You have asked each of your K (2 <= K <= 1000) casting directors to audition N (1 <= N <= 10000) of the actors, and give them each a score (1 <= S_ij <= 10^9). Each director gives you a list of their results, in score order from highest to lowest. (There might be ties.)
- You think it would be most authentic to cast a person with the *median* score. Write a program to find such a person efficiently.

Example:
K = 3, N = 3. Lists are [987654321, 97, 64], [99, 99, 83], [85, 43, 1].
We should cast the first person in the third list.

- My build was working yesterday (at changelist 100000000)
- It's not working now (at changelist 100500000)
- In that interval, someone else's changelist broke it, and it stayed broken
  - so who do I yell at (nicely)?



Binary search to the rescue!

low	high	mid	build OK?
100000000	100500000	100250000	No
100000000	100250000	100125000	Yes
100125000	100250000	100187500	Yes
100187500	100250000	100218750	No
100187500	100218750	100203125	Yes

etc.

Binary search is awesome!

- Takes O(log N) time, where N is the size of the range of values searched
  - Some huge bound like 10^9 on input data is often a clue that the solution involves binary search...

- Also works on a pre-sorted list of values!
  - or you may be able to limit the possible values to a specific set of candidates...

Pixelated Boat
@pixelatedboat

Follow

The whole internet loves **binary search**, a lovely **algorithm that seems easy to code**! *5 seconds later* We regret to inform you the **way you coded it loops forever and is also wrong**

Retweets	Likes
10,146	24,084

1:07 AM - 12 Jun 2016

81 10K 24K

Binary search is awful!

- Infamously easy to mess up when coding it on the spot
  - or even e.g. in CS161 HW with less time pressure…
  - **do not let an interview be the first time you code it up!**

- Only works if values in the range are of the form True True … True False False… or vice versa – i.e a single switch from True to False (or vice versa)

- Example situations where binary search won't work:
  - find largest prime number less than 100000000
  - find minimum of unimodal f(x) on [1, 100000000]

⭐⭐ Problem 4-2: Trendy Dessert

- Some new dessert place just dropped in Palo Alto, and there's a huge line.

- There are K servers (2 <= K <= 1000); the i-th server takes K_i minutes to help a customer, then opens up for the next one. Customers go to the lowest-numbered available server.

- The store just opened. You are position N in line (1 <= N <= 10^9). Which server will help you?

How can we use binary search here?

Guess a time and check

e.g., $K_1 = 20$, $K_2 = 10$, $K_3 = 15$, $N = 10$

assume time in range [0, 20*10] = [0, 200]. Midpoint 100

- 20*10 is a safe upper bound since the slowest server takes 20 minutes

- After 100 minutes:
  - server 1 will have helped ceil(100/20) = 5 customers
  - server 2: ceil(100/10) = 10
  - server 3: ceil(100/15) = 7
  - total customers helped: 22. Too many!

Guess a time and check

e.g., $K_1 = 20$, $K_2 = 10$, $K_3 = 15$, $N = 10$

new range [0, 100]. Midpoint 50

- After 50 minutes:
  - server 1 will have helped ceil(50/20) = 3 customers
  - server 2: ceil(50/10) = 5
  - server 3: ceil(50/15) = 4
  - total customers helped: 12. Still too many!

Guess a time and check

e.g., $K_1 = 20$, $K_2 = 10$, $K_3 = 15$, $N = 10$

new range [0, 50]. Midpoint 25

- After 25 minutes:
    - server 1 will have helped ceil(25/20) = 2 customers
    - server 2: ceil(25/10) = 3
    - server 3: ceil(25/15) = 2
    - total customers helped: 7. Too few!

That's the idea!

- Binary search until you find the time when you get served. Determine which server opened up and served you at that time
- Make sure to handle edge cases where there are ties (lots of other customers served at same instant as you)
- $O(K)$ work per check, $O(\log (NK_{Max}))$ checks

Don't let Ian forget to say the password

(and the solution to 4-3 is on the next slides!)

Average Joe: Solutions

- There are KN actors in total, so we can solve this in O(KN log KN) time by combining all the lists and using an algorithm such as merge sort, then taking the median.

- Or, we can merge the K lists like merge sort does: maintain pointers to the start of every list, then keep identifying and removing a largest score out of those pointed to, until we've seen half the actors. But if we have to look at the starts of all K lists each time to find the current largest score, this is O(K) * KN = $O(K^2N)$...
  - To find the largest score efficiently, we need to use something like a priority queue (remember last week's discussion?) to store the top K elements. Now finding the current maximum is O(log K), which cuts the time to O(KN log K).

- Or, we can combine all the lists and then use a linear-time selection algorithm to solve in O(KN) time. (The details of linear-time median finding are complicated; there's an overview article here: https://rcoh.me/posts/linear-time-median-finding/. This also comes up in CS161.)
  - But is O(KN) a tight bound here? **Do we really need to look at all the data?** 🤔

Average Joe: Best solution?

- There is also a solution sort of like the one in 4-2! In this case it involves a **double** binary search... 😲
    - Outer: Binary search (over the range [1, S]) for the median score S_m.
    - Inner: For each guess, for each of the K lists, binary search on the list to see how many scores are less than S_m. Then take the total of these numbers.
        - If it's less than KN/2, try making S_m bigger.
        - If it's greater than KN/2, try making S_m smaller.
        - If it's KN/2, or if we've found some S_m < KN/2 and S_m + 1 > KN/2, we're done.
- Each check is O(K log N), so the algorithm is **O(K log N log S)**